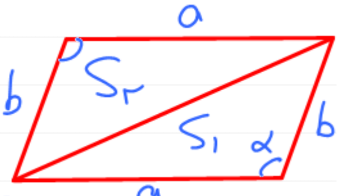
بنام خدا

	۰°	۳۰°	۴۵°	۶۰°	۹۰°
Sin	$\frac{\sqrt{0}}{2}=0$	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}=1$
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	ن ت
cotg	ن ت	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

$$S=\frac{1}{2}a.b.\sin\alpha$$



$$+\begin{cases} S_1=\frac{1}{2}a.b.\sin\alpha \\ S_2=\frac{1}{2}a.b.\sin\alpha \end{cases}$$

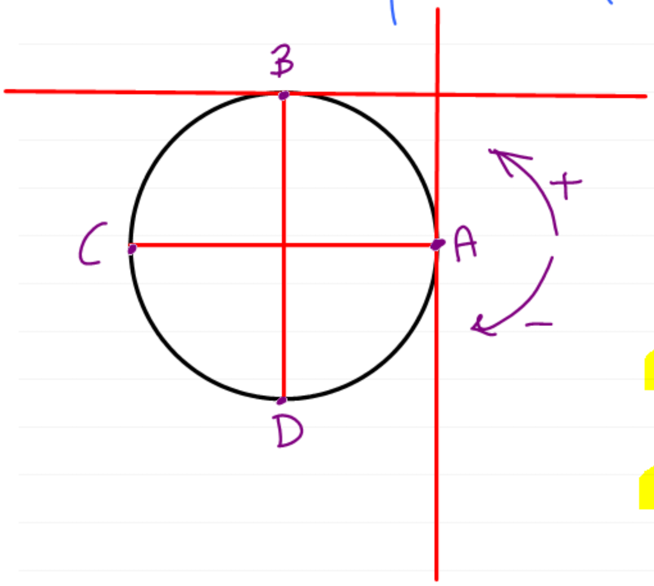
$$\boxed{S_{\text{متوازی‌الاضلاع}}=a.b.\sin\alpha}$$

قانون کسینوس‌ها

$$\begin{cases} a^2=c^2+b^2-2bc\cos\hat{A} \\ b^2=a^2+c^2-2ac\cos\hat{B} \\ c^2=a^2+b^2-2ab\cos\hat{C} \end{cases}$$

قانون سینوس‌ها

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$



$A: 2k\pi$ $\quad$ $B: 2k\pi+\frac{\pi}{2}$

$D: 2k\pi-\frac{\pi}{2}$ $\quad$ $C: 2k\pi+\pi$

$A, C: k\pi$ $\quad$ $B, D: k\pi+\frac{\pi}{2}$

$A, B, C, D: \frac{k\pi}{2}$

$$\sin x = 1 \xrightarrow{B} x = 2k\pi + \frac{\pi}{2}$$

$$\sin(5x) = 1 \xrightarrow{B} 5x = 2k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{5} + \frac{\pi}{10}$$

$$K = 0 \longrightarrow x = \frac{\pi}{10}$$

$$K = 1 \longrightarrow x = \frac{2\pi}{5} + \frac{\pi}{10} = \frac{4\pi}{10} + \frac{\pi}{10} = \frac{5\pi}{10} = \frac{\pi}{2}$$

$\vdots$

$$\sin 4x = 1 \longrightarrow 4x = 2k\pi + \frac{\pi}{2}$$

$$x = \frac{k\pi}{2} + \frac{\pi}{8}$$

$$\sin x = -1 \xrightarrow{D} x = 2k\pi - \frac{\pi}{2}$$

$$\sin 7x = -1 \longrightarrow 7x = 2k\pi - \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{7} - \frac{\pi}{14}$$

$$\sin x = 0 \xrightarrow{A,C} \boxed{x = k\pi}$$

$$\sin 5x = 0 \longrightarrow 5x = k\pi \longrightarrow \boxed{x = \frac{k\pi}{5}}$$

$$\cos x = 0 \xrightarrow{B,D} x = k\pi + \frac{\pi}{2}$$

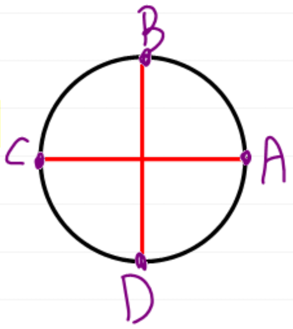
$$\cos 5x = 0 \xrightarrow{B,D} 5x = k\pi + \frac{\pi}{2}$$

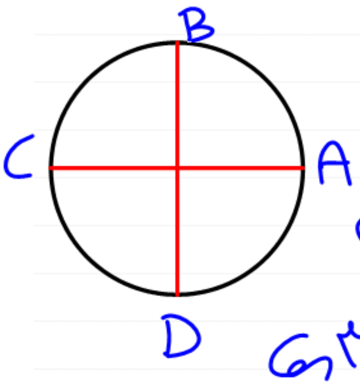
$$\cos x = -1 \xrightarrow{C} x = 2k\pi + \pi$$

$$\cos 4x = 1 \xrightarrow{A} 4x = 2k\pi \longrightarrow x = \frac{k\pi}{2}$$

معادله $\sin x \cos x = 0$ را حل کنید.

$$\left\{\begin{array}{l} \sin x = 0 \xrightarrow{A,C} x = k\pi \\ \cos x = 0 \xrightarrow{B,D} x = k\pi + \frac{\pi}{2} \end{array}\right\} \longrightarrow x = \frac{k\pi}{2}$$





A,C نسبت را عوض نمی‌کنند.

B,D نسبت را عوض می‌کنند.

$$\sin 240^\circ = \sin(180 + 60) = -\sin 60 = -\frac{\sqrt{3}}{2}$$

$$\cos 300 = \cos(270 + 30) = +\sin 30 = \frac{1}{2}$$

$$\left\{\begin{array}{l} \cos 840^\circ = \cos(810 + 30) = -\sin 30 = \frac{-1}{2} \\ \cos 840 = \cos(900 - 60) = -\cos 60 = \frac{-1}{2} \end{array}\right.$$

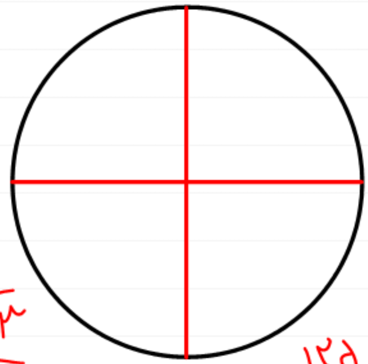
$\cos 570^\circ = \cos(540+30) = -\cos 30 = -\frac{\sqrt{3}}{2}$

$\sin 660^\circ = \sin(630+30) = -\cos 30 = -\frac{\sqrt{3}}{2}$

$\tan 480^\circ = \tan(450+30) = -\cot 30 = -\sqrt{3}$

$\cot 390^\circ = \cot(360+30) = \cot 30 = \sqrt{3}$

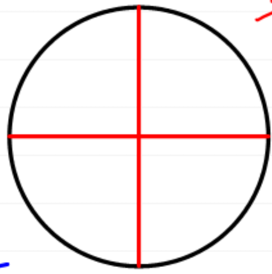
$\tan 330^\circ = \tan(270+60) = -\cot 60 = -\frac{\sqrt{3}}{3}$



$125 \,|\, 4 \to 31$, remainder 1

$\cot \frac{2\pi}{3} = \cot(\pi - \frac{\pi}{3}) = -\cot\frac{\pi}{3} = -\frac{\sqrt{3}}{3}$

$\tan \frac{4\pi}{3} = \tan(\pi + \frac{\pi}{3}) = \tan\frac{\pi}{3} = \sqrt{3}$



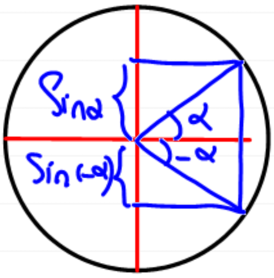
$\cot(\frac{7\pi}{6}) =$

$7 \,|\, 6 \to 1$, remainder 1; $\quad \frac{7\pi}{6} = \pi + \frac{\pi}{6}$

$\sin(\frac{5\pi}{4}) = \sin(\pi + \frac{\pi}{4}) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\cos(\frac{11\pi}{6}) = \cos(2\pi - \frac{\pi}{6}) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sin\frac{125\pi}{4} = \sin(31\pi + \frac{\pi}{4}) = -\sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$



$$\begin{cases} \cos(-\alpha) = \cos\alpha \\ \sin(-\alpha) = -\sin\alpha \\ \tan(-\alpha) = -\tan\alpha \\ \cot(-\alpha) = -\cot\alpha \end{cases}$$

$$\alpha + \beta = 90 \to \begin{cases} \sin\alpha = \cos\beta \\ \tan\alpha = \cot\beta \end{cases}$$

$\cancel{\cos 1^\circ} + \cos 2^\circ + \cdots + \cancel{\cos 179^\circ} = ?$ صفر

$$\alpha + \beta = 180 \longrightarrow \begin{cases} \sin\alpha = \sin\beta \\ \cos\alpha = -\cos\beta \\ \tan\alpha = -\tan\beta \\ \cot\alpha = -\cot\beta \end{cases}$$

$\alpha = 180 - \beta$

$\sin\alpha = \sin(180-\beta) = \sin\beta$

$\alpha + \beta = 180 \to \cos\alpha + \cos\beta = 0$

بنام خدا

$$\begin{cases} a\sin^2 x + b\sin x + c \\ a\cos^2 x + b\cos x + c \end{cases} \Longrightarrow \begin{cases} Max = ? \\ Min = ? \end{cases}$$

$$\begin{cases} \sin x = 1 \\ \sin x = -1 \\ \sin x = 0 \\ -1 \leqslant \sin x = \frac{-b}{2a} \leqslant 1 \end{cases}$$

$$A = 2\sin^2 x - 4\sin x + 7 \Longrightarrow \begin{cases} Max\,A = ? \\ Min\,A = ? \end{cases}$$

$\sin x = 1 \longrightarrow A = 5$

$\sin x = -1 \longrightarrow A = 13$

$\sin x = 0 \longrightarrow A = 7$

$\sin x = \frac{-b}{2a} = \frac{4}{4} = 1 \longrightarrow A = 5$

$$\begin{cases} Max\,A = 13 \\ Min\,A = 5 \end{cases}$$

$$\sin^2\alpha + \cos^2\alpha = 1 \Longrightarrow \begin{cases} \sin^2\alpha = 1 - \cos^2\alpha \\ \cos^2\alpha = 1 - \sin^2\alpha \end{cases}$$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$, $\cot g\alpha = \frac{\cos\alpha}{\sin\alpha}$, $tg\alpha = \frac{1}{\cot g\alpha}$, $\begin{cases} 1 + tg^2\alpha = \frac{1}{\cos^2\alpha} \\ 1 + \cot g^2\alpha = \frac{1}{\sin^2\alpha} \end{cases}$

نکته مهم:

$$\begin{cases} a + \frac{1}{a} \geqslant 2 & a > 0 \\ a + \frac{1}{a} \leqslant -2 & a < 0 \end{cases}$$

$a + \frac{1}{a} = 2 \Longleftrightarrow \boxed{a = 1}$

برد تابع $f(x) = x^2 + \frac{1}{x^2}$ کدام است؟

$R_f = [2, +\infty)$

$4, 9$ میانگین حسابی $= 6.5$

میانگین هندسی $= \sqrt{4 \times 9} = 6$

a, b دو عدد مثبت حقیقی

$$\boxed{\frac{a+b}{2} \geqslant \sqrt{ab}} \longrightarrow a + b \geqslant 2\sqrt{ab}$$

برد تابع $f(x) = x^2 + \frac{9}{x^2}$ را بیابید.

$a + b \geqslant 2\sqrt{ab}$

$f(x) = x^2 + \frac{9}{x^2} \geqslant 2\sqrt{x^2 \times \frac{9}{x^2}} = 6$

$R_f = [6, +\infty)$

برد تابع $f(x)=[\tan x+\cot x]$ شامل چند عدد صحیح نمی‌باشد؟

$$\begin{cases}\tan x+\cot x\geq 2\\ \tan x+\cot x\leq -2\end{cases}\longrightarrow\begin{cases}y=2,3,4,5,\ldots\\ y=-2,-3,-4,-5,\ldots\end{cases}$$

برد فقط شامل اعداد صحیح 1 ، 0 ، -1 نمی‌باشد.

$$\sin^4\alpha+\cos^4\alpha=(\sin^2\alpha+\cos^2\alpha)^2-2\sin^2\alpha\cos^2\alpha=1-2\sin^2\alpha\cos^2\alpha$$

$$\sin^6\alpha+\cos^6\alpha=\underbrace{(\sin^2\alpha+\cos^2\alpha)}_{\text{یک}}(\sin^4\alpha-\sin^2\alpha\cos^2\alpha+\cos^4\alpha)=1-3\sin^2\alpha\cos^2\alpha$$

بنام خدا

روابط شاتی 2α

$$\sin 2\alpha=2\sin\alpha\cos\alpha\Longrightarrow\sin\alpha\cos\alpha=\frac{1}{2}\sin 2\alpha$$

$$\begin{cases}\sin 70^\circ=2\sin 35^\circ\cos 35^\circ\\ \sin 50^\circ=2\sin 25^\circ\cos 25^\circ\\ \sin 4x=2\sin 2x\cos 2x\\ \sin 3x=2\sin\frac{3}{2}x\cos\frac{3}{2}x\\ \sin x=2\sin\frac{x}{2}\cos\frac{x}{2}\end{cases}$$

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$$

$$\cos 2\alpha=1-2\sin^2\alpha$$

$$\cos 2\alpha=2\cos^2\alpha-1$$

اگر $f(x)=\sin x\cos^3 x-\sin^3 x\cos x$ باشد $f(\frac{\pi}{24})$ را بیابید.

$$f(x)=\underbrace{\sin x\cos x}(\underbrace{\cos^2 x-\sin^2 x})$$

$$=\frac{1}{2}\sin 2x\cos 2x=\frac{1}{2}\left(\frac{1}{2}\sin 4x\right)=\frac{1}{4}\sin 4x$$

$$f\left(\frac{\pi}{24}\right)=\frac{1}{4}\times\sin\frac{\pi}{6}=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$$

حاصل عبارت $\underbrace{\sin x\cos x}\,\underbrace{(1-2\sin^2 x)}$ را به ازای $x=7.5^\circ$ بیابید.

$$=\frac{1}{2}\sin 2x\cos 2x$$

$$=\frac{1}{2}\left(\frac{1}{2}\sin 4x\right)=\frac{1}{4}\sin 4x\Big|_{x=7.5^\circ}=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$$

اگر $\cos x = \frac{1}{3}$ باشد حاصل $\cos 4x$ را بیابید.

$$\cos x = \frac{1}{3} \longrightarrow \cos 2x = 2\cos^2 x - 1 \Longrightarrow \cos 2x = 2\left(\frac{1}{9}\right) - 1 = \frac{-7}{9}$$

$$\cos 4x = 2\cos^2 2x - 1 = 2\left(\frac{49}{81}\right) - 1 = \frac{17}{81}$$

$t = \sin 22.5^\circ = ?$ $\qquad \cos 45^\circ = 1 - 2\sin^2 22.5^\circ$

$$\frac{\sqrt{2}}{2} = 1 - 2t^2 \longrightarrow 2t^2 = 1 - \frac{\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$$

$$t^2 = \frac{2-\sqrt{2}}{4} \longrightarrow t = \frac{\sqrt{2-\sqrt{2}}}{2}$$

$$\begin{cases} \cos 2\alpha = 2\cos^2\alpha - 1 \longrightarrow \cos^2\alpha = \dfrac{1+\cos 2\alpha}{2} \\ \cos 2\alpha = 1 - 2\sin^2\alpha \longrightarrow \sin^2\alpha = \dfrac{1-\cos 2\alpha}{2} \end{cases}$$

$$\sin^2 22.5 = \frac{1-\cos 45}{2} = \frac{1-\frac{\sqrt{2}}{2}}{2} = \frac{2-\sqrt{2}}{4} \longrightarrow \sin 22.5 = \frac{\sqrt{2-\sqrt{2}}}{2}$$

$$\cos 15^\circ = ? \qquad \cos^2 15^\circ = \frac{1+\cos 30^\circ}{2} = \frac{1+\frac{\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{4} \to \cos 15^\circ = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$$\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = \frac{2}{2\sin x \cos x}$$

$$\tan x + \cot x = \frac{2}{\sin 2x}$$

$$\tan x - \cot x = \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} = \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} = \frac{-\cos 2x}{\frac{1}{2}\sin 2x} = -2\cot 2x$$

$$\tan x - \cot x = -2\cot 2x$$

$$(\sin x \pm \cos x)^2 = 1 \pm \sin 2x$$

$$\underbrace{\sin^2 x + \cos^2 x}_{1} + 2\sin x \cos x = 1 + \sin 2x$$

$$\sin^4\alpha + \cos^4\alpha = 1 - 2\sin^2\alpha\cos^2\alpha = 1 - 2(\sin\alpha\cos\alpha)^2$$

$$1 - 2\left(\frac{1}{2}\sin 2\alpha\right) = 1 - \frac{1}{2}\sin^2 2\alpha$$

$$\sin^4\alpha + \cos^4\alpha = 1 - \frac{1}{2}\sin^2 2\alpha$$

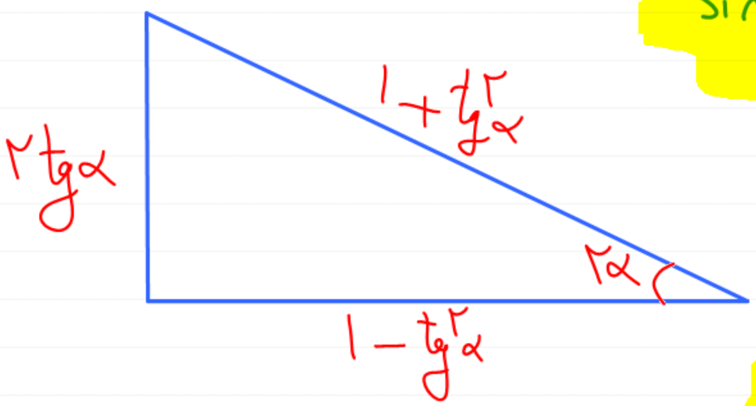
$$\sin^6\alpha + \cos^6\alpha = 1 - \frac{3}{4}\sin^2 2\alpha$$

حاصل $\sin^4\frac{\pi}{12}+\cos^4\frac{\pi}{12}$ را بیابید.

$$\sin^4\frac{\pi}{12}+\cos^4\frac{\pi}{12}=1-\frac{3}{4}\sin^2\left(\frac{\pi}{6}\right)=1-\frac{3}{4}\times\frac{1}{4}=\frac{13}{16}$$

حاصل $\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}$ را بیابید.

$$=1-\frac{1}{2}\sin^2\left(\frac{\pi}{4}\right)=1-\frac{1}{2}\times\left(\frac{\sqrt{2}}{2}\right)^2=\frac{3}{4}$$



$$\sin2\alpha=\frac{2\tan\alpha}{1+\tan^2\alpha}$$

$$\cos2\alpha=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}$$

$$\tan2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$$

اگر $f(x)=\frac{\tan x(1-\tan^2x)}{(1+\tan^2x)^2}$ باشد $f(\frac{\pi}{24})$ را بیابید.

$$f(x)=\frac{2\tan x}{1+\tan^2x}\times\frac{1-\tan^2x}{1+\tan^2x}\times\frac{1}{2}$$

$$f(x)=\sin2\alpha\cos2\alpha\times\frac{1}{2}=\frac{1}{4}\sin4x$$

$$f\left(\frac{\pi}{24}\right)=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$$

نسبت‌های مثلثاتی $\alpha \pm \beta$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

$$\sin 75 = \sin(45+30) = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\cos 105^\circ = \cos(45+60) = \cos 45\cos 60 - \sin 45\sin 60 = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$$

* اگر $\tan(x+y)=3$, $\tan(x-y)=2$ باشد حاصل $\tan 2x$ را بیابید.

$$\tan(2x) = \tan[(x+y)+(x-y)] = \frac{3+2}{1-(3)(2)} = \frac{5}{-5} = -1$$

مثال: $\tan 29 + \tan 31 + \sqrt{3}\tan 29\tan 31 = ?$

$$\tan(29+31) = \tan 60$$

$$\frac{\tan 29 + \tan 31}{1 - \tan 29\tan 31} = \sqrt{3} \Longrightarrow \tan 29 + \tan 31 = \sqrt{3} - \sqrt{3}\tan 29\tan 31$$

$$\tan 29 + \tan 31 + \sqrt{3}\tan 29\tan 31 = \sqrt{3}$$

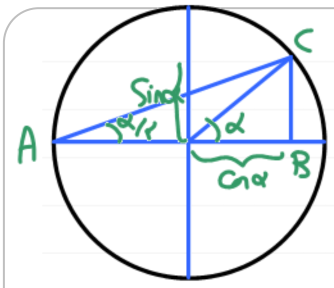
$$\sqrt{2}\sin\left(x+\frac{\pi}{4}\right) = ?$$

$$= \sqrt{2}\left(\sin x\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos x\right)$$

$$= \sqrt{2}\left(\frac{\sqrt{2}}{2}\right)(\sin x + \cos x) = \sin x + \cos x$$

$$\sin x \pm \cos x = \sqrt{2}\sin\left(x \pm \frac{\pi}{4}\right)$$

مثال: $\sin\frac{\pi}{12} - \cos\frac{\pi}{12} = \sqrt{2}\sin\left(\frac{\pi}{12} - \frac{\pi}{4}\right) = \sqrt{2}\sin\left(-\frac{\pi}{6}\right) = -\sqrt{2}\times\frac{1}{2} = -\frac{\sqrt{2}}{2}$



$$\tan\frac{\alpha}{2} = \frac{BC}{AB} = \frac{\sin\alpha}{1+\cos\alpha}$$

$$\tan 22{,}5 = \frac{\sin 45}{1+\cos 45} = \frac{\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{2\sqrt{2}-2}{2} = \sqrt{2}-1$$

طلائی

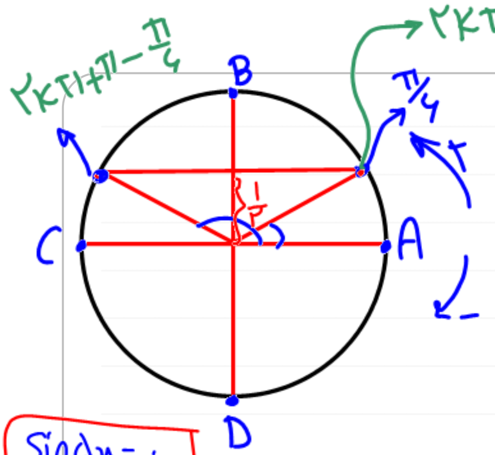
$$\begin{cases} \sin^2\alpha = \dfrac{1-\cos 2\alpha}{2} \\ \cos^2\alpha = \dfrac{1+\cos 2\alpha}{2} \end{cases} \Longrightarrow \tan^2\alpha = \frac{1-\cos 2\alpha}{1+\cos 2\alpha}$$

اینو برعکس

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1-\cos 2\alpha = 2\sin^2\alpha \end{cases}$$

ثابت کنید: $\dfrac{\sin\alpha}{1+\cos\alpha} = \tan\dfrac{\alpha}{2}$

$$\frac{\sin\alpha}{1+\cos\alpha} = \frac{\cancel{2}\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cancel{2}\cos^2\frac{\alpha}{2}} = \tan\frac{\alpha}{2}$$



$A: 2k\pi$ $\quad$ $B: 2k\pi + \frac{\pi}{2}$

$D: 2k\pi - \frac{\pi}{2}$ $\quad$ $C: 2k\pi + \pi$

$A, C: k\pi$ $\quad$ $B, D: k\pi + \frac{\pi}{2}$

$A, B, C, D: \frac{k\pi}{2}$

$(k \in \mathbb{Z})$

$\sin 5x = 0$
$5x = k\pi$
$\boxed{x = \frac{k\pi}{5}}$

$\cos 3x = 0$
$3x = k\pi + \frac{\pi}{2}$
$x = \frac{k\pi}{3} + \frac{\pi}{6}$

$$\sin x = \frac{1}{2} \Longrightarrow \sin x = \sin \alpha = \frac{1}{2}$$

$$\xrightarrow{\alpha = \frac{\pi}{6}} \begin{cases} x = 2k\pi + \alpha = 2k\pi + \frac{\pi}{6} \\ x = 2k\pi + \pi - \alpha = 2k\pi + \pi - \frac{\pi}{6} \end{cases}$$

$$2\sin x = \sqrt{3} \longrightarrow \sin x = \frac{\sqrt{3}}{2} \xrightarrow{\alpha = \frac{\pi}{3}} \begin{cases} x = 2k\pi + \frac{\pi}{3} \\ x = 2k\pi + \pi - \frac{\pi}{3} = 2k\pi + \frac{2\pi}{3} \end{cases}$$

معادله $\underbrace{\cos 2x}$ $+ 2\sin x - 1 = 0$ را حل کنید.

$$1 - 2\sin^2 x + 2\sin x - 1 = 0$$

$$-2\sin^2 x + 2\sin x = 0$$

$$-2\sin x(\sin x - 1) = 0$$

$$\begin{cases} \sin x = 0 \xrightarrow{A, C} x = k\pi \\ \sin x = 1 \xrightarrow{B} x = 2k\pi + \frac{\pi}{2} \end{cases}$$

$$\alpha = \frac{\pi}{2} \rightarrow \begin{cases} x = 2k\pi + \frac{\pi}{2} \\ x = 2k\pi + \pi - \frac{\pi}{2} \end{cases}$$

$$\begin{cases} \cos 2x = \cos^2 x - \sin^2 x \\ \cos 2x = 1 - 2\sin^2 x \\ \cos 2x = 2\cos^2 x - 1 \end{cases}$$

$*$ $\underbrace{\sin 3x}_{X} = \underbrace{\sin 2x}_{\alpha}$

$$\begin{cases} 3x = 2k\pi + 2x \longrightarrow x = 2k\pi \\ 3x = 2k\pi + \pi - 2x \longrightarrow 5x = 2k\pi + \pi \\ \qquad x = \frac{2k\pi}{5} + \frac{\pi}{5} \end{cases}$$

$*$ $\sin 3x = \cos 2x$

$$\sin 3x = \sin\left(\frac{\pi}{2} - 2x\right) \longrightarrow \begin{cases} 3x = 2k\pi + \left(\frac{\pi}{2} - 2x\right) \\ 3x = 2k\pi + \pi - \left(\frac{\pi}{2} - 2x\right) \end{cases}$$

$$\begin{cases} 5x = 2k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{5} + \frac{\pi}{10} \\ x = 2k\pi + \frac{\pi}{2} \end{cases}$$

$* \quad \sin x \cos x = \frac{\sqrt{2}}{4}$

$$\frac{1}{2}\sin 2x = \frac{\sqrt{2}}{4} \Longrightarrow \sin 2x = \frac{\sqrt{2}}{2} \xrightarrow{\alpha=\frac{\pi}{4}} \begin{cases} 2x = 2k\pi + \frac{\pi}{4} \\ 2x = 2k\pi + \pi - \frac{\pi}{4} \end{cases}$$

$$\begin{cases} x = k\pi + \frac{\pi}{8} \\ x = k\pi + \frac{3\pi}{8} \end{cases}$$

k	-1	0	1	2
x	منفی	$\frac{\pi}{8}$, $\frac{3\pi}{8}$	$\frac{9\pi}{8}$, $\frac{11\pi}{8}$	✗ بالاتر از 2π

$[0, 2\pi]$

$$\sin x \pm \cos x = \sqrt{2}\sin\left(x \pm \frac{\pi}{4}\right)$$

(کتاب درسی) معادله $\underbrace{\sin x + \cos x = 1}$ را حل کنید

$$\sqrt{2}\sin\left(x+\frac{\pi}{4}\right) = 1 \longrightarrow \underbrace{\sin\left(x+\frac{\pi}{4}\right)}_{x} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \sin\underbrace{\frac{\pi}{4}}_{\alpha}$$

$\alpha = \frac{\pi}{4}$

$$\begin{cases} x + \frac{\pi}{4} = 2k\pi + \frac{\pi}{4} \longrightarrow x = 2k\pi \\ x + \frac{\pi}{4} = 2k\pi + \pi - \frac{\pi}{4} \longrightarrow x = 2k\pi + \frac{\pi}{2} \end{cases}$$

$* \quad \sin 3x + \sin 2x = 0 \longrightarrow \sin 3x = -\sin 2x$

$$\sin\underbrace{3x}_{x} = \sin\underbrace{(-2x)}_{\alpha}$$

$$\begin{cases} 3x = 2k\pi + (-2x) \longrightarrow x = \frac{2k\pi}{5} \\ 3x = 2k\pi + \pi - (-2x) \longrightarrow x = 2k\pi + \pi \end{cases}$$

جواب کلی معادله $\frac{\sin 3x + \sin 2x}{1 + \cos x} = 0$ کدام است؟

$\frac{2k\pi}{5}$ ✓

$\frac{k\pi}{5}$

✗ $2k\pi + \pi$

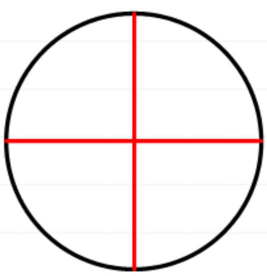
$2k\pi + \frac{\pi}{5}$

بالا حل شد $\sin 3x + \sin 2x = 0$

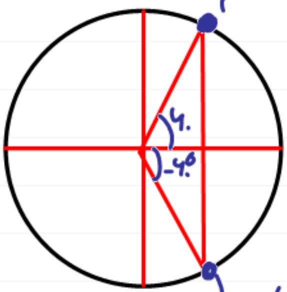
$\sin 3x = -\sin 2x = \sin(-2x)$

$$\begin{cases} x = \frac{2k\pi}{5} \\ x = 2k\pi + \pi \end{cases}$$

قبول نیست؛ ریشه مخرج است.



برخط فهمیده



$\cos x=\frac{1}{2} \longrightarrow \cos x=\cos\alpha=\frac{1}{2}$

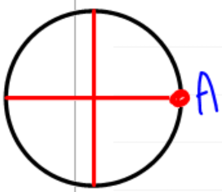
$\begin{cases} x=2k\pi+\alpha \\ x=2k\pi-\alpha \end{cases}$

$\begin{cases} x=2k\pi+\frac{\pi}{3} \\ x=2k\pi-\frac{\pi}{3} \end{cases}$

$\cos 2x+\cos x-2=0$

$2\cos^2 x-1+\cos x-2=0$

$2\cos^2 x+\cos x-3=0 \xrightarrow{\cos x=t} 2t^2+t-3=0 \longrightarrow \begin{cases} t=1 \\ t=\frac{-3}{2} \end{cases}$

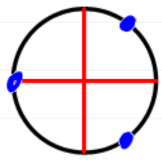


$\begin{cases} \cos x=1 \xrightarrow{A} \boxed{x=2k\pi} \\ \cos x=\frac{-3}{2} \; \times \end{cases}$

$\cos 2x=-\cos x$

$\underbrace{\cos 2x}_{x}=\cos(\underbrace{\pi-x}_{\alpha}) \Longrightarrow \begin{cases} 2x=2k\pi+(\pi-x) \\ 2x=2k\pi-(\pi-x) \end{cases}$

$\begin{cases} 3x=2k\pi+\pi \longrightarrow \boxed{x=\frac{2k\pi}{3}+\frac{\pi}{3}} \\ \boxed{x=2k\pi-\pi} \end{cases}$



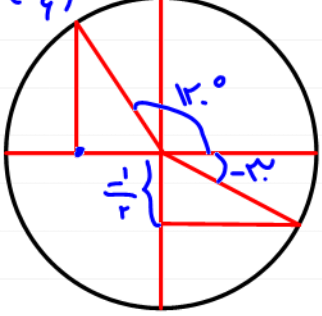
(روش دوم: $2\cos^2 x-1+\cos x=0 \longrightarrow 2\cos^2 x+\cos x-1=0$

$\cos x=-1 \longrightarrow x=2k\pi+\pi$

$\cos x=\frac{1}{2} \longrightarrow x=2k\pi\pm\frac{\pi}{3}$

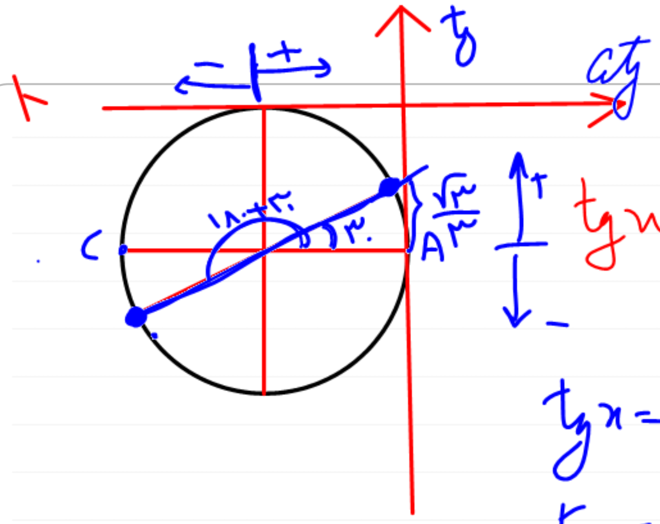
(ربع چهارم) $\sin x=\frac{-1}{2} \xrightarrow{\alpha=\frac{-\pi}{6}} \begin{cases} x=2k\pi+(-\frac{\pi}{6}) \\ x=2k\pi+\pi-(\frac{-\pi}{6}) \end{cases}$

(ربع دوم) $\cos x=-\frac{1}{2} \xrightarrow{\alpha=\frac{2\pi}{3}} \begin{cases} x=2k\pi+\frac{2\pi}{3} \\ x=2k\pi-\frac{2\pi}{3} \end{cases}$



$\tan x=-\frac{\sqrt{3}}{3} \xrightarrow{\alpha=\frac{-\pi}{6}}$

(ربع چهارم)



$tg\,x = \frac{\sqrt{3}}{3} \xrightarrow{\alpha=\frac{\pi}{6}} \boxed{x = k\pi+\alpha = k\pi+\frac{\pi}{6}}$

$tg\,x = 1 \xrightarrow{\alpha=\frac{\pi}{4}} x = k\pi + \frac{\pi}{4}$

$tg\,x = \sqrt{3} \xrightarrow{\alpha=\frac{\pi}{3}} x = k\pi + \frac{\pi}{3}$

$tg\,x = -1 \xrightarrow{\alpha=-\frac{\pi}{4}} x = k\pi + \left(-\frac{\pi}{4}\right)$

$tg\,3x + tg\,2x = 0$

$\underbrace{tg\,3x}_{x} = -tg\,2x = tg\underbrace{(-2x)}_{\alpha} \longrightarrow 3x = k\pi + (-2x)$

$5x = k\pi$

$\boxed{x = \frac{k\pi}{5}}$

$a>0$ $\quad a+\frac{1}{a} \geq 2$

$a=1$

$a+\frac{1}{a}=2$

$tg\,x + cotg\,x = 2$

$\tan x + \frac{1}{\tan x} = 2 \longrightarrow tg^2x + 1 = 2\,tg\,x \longrightarrow \tan^2 x - 2\,tg\,x + 1 = 0$

$(tg\,x - 1)^2 = 0 \longrightarrow tg\,x = 1 \xrightarrow[\alpha=\frac{\pi}{4}]{} x = k\pi + \frac{\pi}{4}$

$tg\,3x = cotg\,2x$

$tg\,3x = tg\left(\frac{\pi}{2} - 2x\right) \longrightarrow 3x = k\pi + \left(\frac{\pi}{2} - 2x\right)$

$5x = k\pi + \frac{\pi}{2}$

$\boxed{x = \frac{k\pi}{5} + \frac{\pi}{10}}$

$\tan 3x \; tg\,2x = 1$

$\tan 3x = \frac{1}{\tan 2x} = cotg\,2x = \tan\left(\frac{\pi}{2} - 2x\right) \longrightarrow 3x = k\pi + \frac{\pi}{2} - 2x$

$x = \frac{k\pi}{5} + \frac{\pi}{10}$

برخط فهمیده

بنام خدا

تابع متناوب

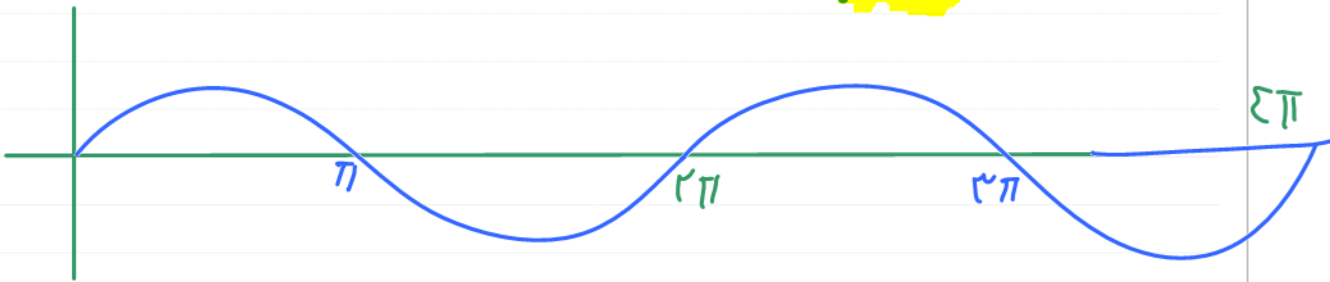
$$\begin{cases} x \in D_f \Rightarrow (x+T) \in D_f \\ f(x+T) = f(x) \end{cases} \longrightarrow$$ T را دوره تناوب می‌گوییم

$f(x) = \sin x$

$T = 2\pi \qquad f(x+2\pi) = \sin(x+2\pi) = \sin x = f(x)$

$T = 4\pi \qquad f(x+4\pi) = \sin(x+4\pi) = \sin x = f(x)$

$T > 0$ که کوچکترین باشد و موجود باشد را دوره تناوب اصلی یک تابع می‌نامیم.

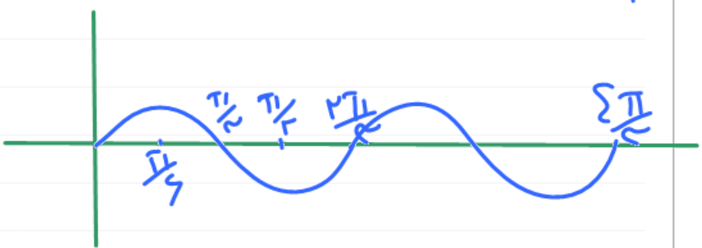


$f(x) = \cos x$

$T = 2\pi \qquad f(x+2\pi) = \cos(x+2\pi) = \cos x = f(x)$

$$\left.\begin{matrix} f(x) = \sin(bx) \\ f(x) = \cos(bx) \end{matrix}\right\} \longrightarrow T = \frac{2\pi}{|b|}$$

$f(x) = \sin 3x \longrightarrow T = \frac{2\pi}{3}$



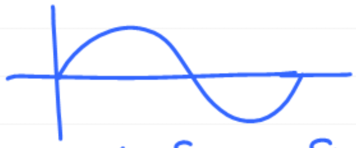
$f(x) = \sin\frac{3}{4}x \longrightarrow T = \frac{2\pi}{\left|\frac{3}{4}\right|} = \frac{8\pi}{3}$

$f(x) = tg\frac{4\pi}{3}x \longrightarrow T = \frac{\pi}{\frac{4\pi}{3}} = \frac{3}{4}$

$$\left.\begin{matrix} f(x) = tg(bx) \\ f(x) = cotg(bx) \end{matrix}\right\} \longrightarrow T = \frac{\pi}{|b|}$$

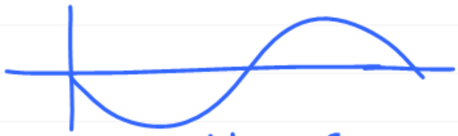
$f(x)= a\sin(bx)+c$

۱)



$y=\sin x=-\sin(-x)$

$ab>0$



$y=-\sin x=\sin(-x)$

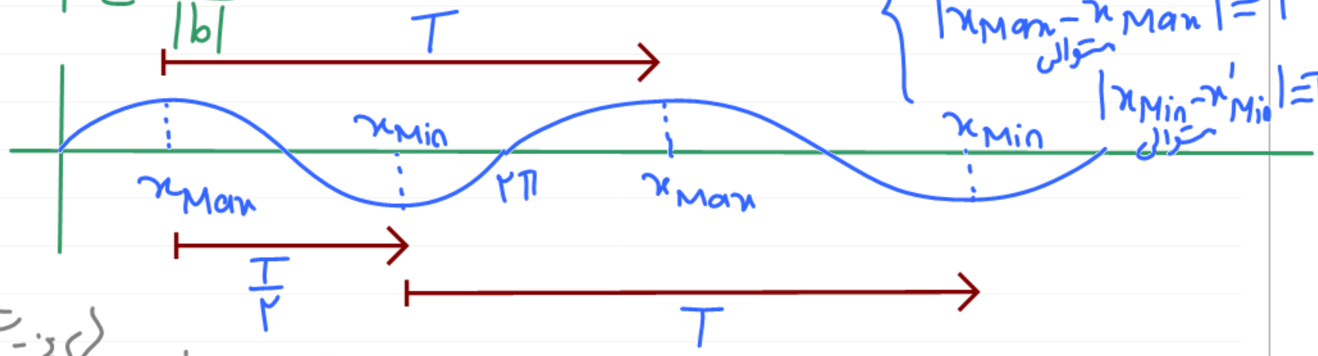
$ab<0$

۲) $\begin{cases} \operatorname{Max} f=|a|+c \\ \operatorname{Min} f=-|a|+c \end{cases} \longrightarrow c=\dfrac{\operatorname{Max} f+\operatorname{Min} f}{2}$

۳) $T=\dfrac{2\pi}{|b|}$

$\begin{cases} |\underbrace{x_{Max}-x_{Min}}_{\text{متوالی}}|=\dfrac{T}{2} \\ |\underbrace{x_{Max}-x'_{Max}}_{\text{متوالی}}|=T \\ |\underbrace{x_{Min}-x'_{Min}}_{\text{متوالی}}|=T \end{cases}$

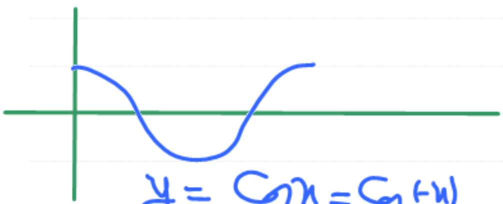
۴)



۵) نقاط تقاطع
به محل برخورد منحنی با محورها دقت کنید.

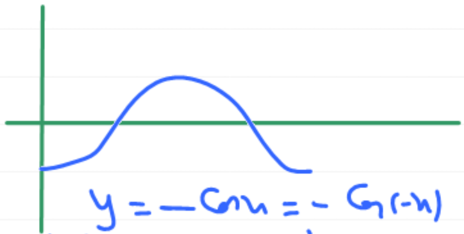
$y=a\cos(bx)+c$

$\cos(-\alpha)=\cos\alpha$



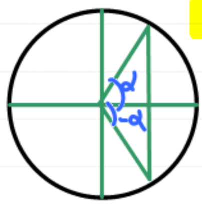
$y=\cos x=\cos(-x)$

$a>0$



$y=-\cos x=-\cos(-x)$

$a<0$



$\cos(\frac{\pi}{3})=\cos(-\frac{\pi}{3})$

b هم مثبت و هم منفی هر دو قابل قبول است.

نکات ۲،۳،۴،۵ با سینوس مشترک